BOOK OF ABSTRACTS

Some embeddings and applications

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Let $p \in (1, \infty)$ and let B_1^c be the exterior of the closed unit ball in $\mathbb{R}^N$. Depending on the dimension N and the values of p, we discuss some embeddings of the Sobolev space $W^{1,p}(B_1^c)$ into certain Lorentz space or, weighted Lebesgue spaces. As an application, we consider the weighted Neumann eigenvalue problem.

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p-harmonic Green functions and their local integrability

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A *p-harmonic function* on weighted $\mathbb{R}^n$, equipped with a measure $d\mu = w\,dx$, is a continuous weak solution u of

$$\operatorname{div}(w|\nabla u|^{p-2}\nabla u) = 0.$$

On metric spaces *p-harmonic functions* are defined as minimizers u of the p-energy integral

$$\int g_u^p \, d\mu,$$

where g_u is the so-called minimal (p-weak) upper gradient of u.

In this talk I will discuss how one can define p-harmonic Green functions on metric spaces, with respect to bounded domains. I will also discuss uniqueness, and the local integrability of the Green functions and their gradients. The results are new even in the linear case on weighted $\mathbb{R}^n$, and make it possible to obtain sharp local integrability results for linear superharmonic functions on weighted $\mathbb{R}^n$.

This talk is based on joint work with Jana Björn and Juha Lehrbäck.

Boundary regularity for nonlinear parabolic equations

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The p-parabolic equation

$$\partial_t u = \Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u), \quad p > 1,$$

is a nonlinear cousin of the classical heat equation. As such, it offers both difficulties and advantages compared with the heat equation. In the talk, we consider the Perron method for solving the Dirichlet problem for the p-parabolic equation in general bounded domains in $\mathbf{R}^{n+1}$. Compared to space-time cylinders, such domains allow the space domain to change in time.

Of particular interest will be boundary regularity for such domains, i.e. whether solutions attain their boundary data in a continuous way. Relations between regular boundary points and barriers will be discussed, as well as some peculiar examples and surprising phenomena related to boundary regularity.

If time permits, the normalized p-parabolic equation $\partial_t u = |\nabla u|^{2-p}\Delta_p u$, as well as the porous medium equation $\partial_t u = \Delta(u^m)$, with surprisingly different behaviour, will also be discussed.

The talk is based on collaborations with Anders Björn (Linköping), Ugo Gianazza (Pavia), Mikko Parviainen (Jyväskylä) and Juhana Siljander (Chicago).

Semilinear Dirichlet problem for the fractional Laplacian

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We give a general framework and results on existence, representation, and uniqueness of solutions to the following semilinear problem

$$-\Delta^{\alpha/2}u(x) = F(x, u(x)), \quad x \in D,$$

with Dirichlet conditions on the complement and at the boundary of general open $D \subset \mathbf{R}^d$. The important special case is $F(x, u(x)) = |u(x)|^\beta$, which raises the question how large β can be. This is a joint work with Sven Jarohs (Frankfurt) and Edyta Kania (Wrocław).

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Stochastic differential equations via generalized ODEs

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The theory of generalized ordinary differential equations lies on the fact that these equations encompass various types of other equations such as ordinary differential equations, impulsive differential equations, measure differential equations, functional differential equations and dynamic equations on time scales.

In this talk, we present the theory of stochastic differential equations via the theory of generalized ordinary differential equations. We show that under some conditions, the Itô integral can be considered as a Kurzweil integral.

Optimal space of Hardy potentials

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For $p \in (1, N)$ and $\Omega \subseteq \mathbb{R}^N$ open, the Beppo-Levi space $\mathcal{D}_0^{1,p}(\Omega)$ is the completion of $C_c^\infty(\Omega)$ with respect to the norm $\|u\| := \left(\int_\Omega |\nabla u|^p\right)^{\frac{1}{p}}$. Using the p-capacity, we define a norm and then identify the optimal Banach function space $\mathcal{H}(\Omega)$ with the set of all $g \in L^1_{loc}(\Omega)$ that admits the following Hardy-Sobolev type inequality:

$$\int_\Omega |g||u|^p \le C \int_\Omega |\nabla u|^p, \ ,$$

for some $C > 0$. Further, we characterize the set of all $g \in \mathcal{H}(\Omega)$ for which the map $G(u) = \int_\Omega g|u|^p$ is compact on $\mathcal{D}_0^{1,p}(\Omega)$. As a consequence, we prove an embedding of $\mathcal{D}_0^{1,p}(\Omega)$ which is finer than the Lorentz-Sobolev embedding.

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Waves of maximal height for a class nonlocal equations with homogeneous symbol

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We discuss the existence and regularity of periodic traveling wave solutions of a class of nonlocal equations with homogeneous symbol of order $-r$, where $r > 1$. Based on the properties of the nonlocal convolution operator, we apply analytic bifurcation theory and show that a highest, peaked periodic traveling wave solution is reached as the limiting case at the end of the main bifurcation curve. The regularity of the highest wave is proved to be exactly Lipschitz. As an application of our analysis, we reformulate the steady reduced Ostrovsky equation in a nonlocal form in terms of a Fourier multiplier operator with symbol $m(k) = k^{-2}$. Thereby we recover its unique highest 2π-periodic, peaked traveling wave solution, having the property of being exactly Lipschitz at the crest.

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Improved compactness in Hajłasz spaces

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On Riemannian manifolds compact Sobolev embedding could be improved in the presence of symmetries. We would like to investigate if such kind of results holds for Sobolev spaces defined on metric measure space.

It turns out, that using concentration compactness of P.L.Lions one can obtain compact embedding $M^{1,p}_H(X) \hookrightarrow\hookrightarrow L^p(X)$, where $M^{1,p}_H$ is space of functions from Hajłasz-Sobolev space invariant under the action of group H. One of the most interesting facts is that a group H is a subgroup of measure preserving group. In my talk I would like to show how one can obtain such improvement in Hajasz spaces.

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Operators in generalized Morrey-type spaces and one dimensional inequalities involving Hardy operators

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In this talk I will present main tools which allowed us to reduce the problem of the boundedness of many operators from Harmonic analysis in Generalized Morrey-type spaces to the one dimensional inequalities involving Hardy operators and its iterations. We give characterization such inequalities.

On pointwise estimates

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We will discuss the validity of some pointwise estimates for functions defined in irregular domains in the Euclidean n-space. As an application we will show that there exist embeddings into suitable Orlicz spaces from the L^1_p-space such that the corresponding Orlicz norm depends on the geometry of the given domain.

My talk is based on joint work with Petteri Harjulehto.

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Solving variational-hemivariational inequalities by application of nonsmooth optimization

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In the presentation we numerically approximate a solution to a class of time dependent mechanical contact problems leading to variational-hemivariational inequalities. We introduce an abstract nonsmooth optimization problem and prove existence of unique solution. Next, we present a fully discrete numerical scheme approximating the solution and provide numerical error estimation. We apply the abstract theory to a time dependent contact problem describing viscoelastic body in contact with a foundation. This contact is governed by a non-monotone nonsmooth friction law with dependence on normal and tangential components of velocity. Weak formulation of introduced contact problem leads to a variational-hemivariational inequality. Finally, we show results of computational simulations, describe the numerical algorithm that is used to obtain these results and present analysis of convergence order. This presentation is a joint work with A. Ochal.

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MHD equations in a bounded domain

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We consider the Dirichlet boundary value problem for the incompressible magnetohydrodynamical (MHD) system

$$\begin{aligned}
&u_t - \nu\Delta u + u\cdot\nabla u = -\nabla p + b\cdot\nabla b, \quad x\in\Omega\subset\mathbb{R}^N,\ t>0,\\
&b_t - \eta\Delta b + u\cdot\nabla b = b\cdot\nabla u, \quad x\in\Omega\subset\mathbb{R}^N,\ t>0,\\
&\operatorname{div}\, u = \operatorname{div}\, v = 0,\\
&u = 0, \quad b = 0 \quad \text{on } \partial\Omega,\\
&u(0,x) = u_0(x), \quad b(0,x) = b_0(x), \quad x\in\Omega,
\end{aligned}$$

in a bounded domain $\Omega\subset\mathbb{R}^N$ with C^2 boundary, where $N = 2,3$. Here u is the velocity of the fluid flow and b is the magnetic field. These functions are the vector-valued functions of $x\in\Omega$ and $t\geq 0$ ($u(t,x) = (u_1(t,x),\dots,u_N(t,x))$, $b(t,x) = (b_1(t,x),\dots,b_N(t,x))$). The total pressure $p = p(t,x)$ is real-valued function of $x\in\Omega$ and $t\geq 0$. The constant $\nu>0$ is the viscosity of the fluid and $\eta>0$ is the magnetic diffusivity.
Using Dan Henry's semigroup approach and Giga-Miyakawa estimates we construct global in time, unique solution to fractional approximations of the MHD system in the base space $(L^2(\Omega))^N\times(L^2(\Omega))^N$. Solution to MHD system is obtained next as a limits of that fractional approximations.

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Mean value property for solutions of PDEs

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During my talk I will compare functions with the mean value property (MVP) with solutions to PDEs. I will give an insight into three notions of MVP: strong, weak and asymptotic. Moreover, I will discuss PDEs such that their solution possess MVP. In the end, I will present a PDE characterization of functions which possess MVP, when the underlying space is equipped with a norm induced metric and a weighted Lebsegue measure.

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Optimal design for thin structures in generalized Sobolev spaces

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We deal with the problem

$$\begin{aligned}\inf_{\substack{v\,\in\, W^{1,\Psi}(\Omega(\varepsilon);\mathbb{R}^3)\\ \chi_{E(\varepsilon)}\,\in\, BV(\Omega(\varepsilon);\{0,1\})}} \quad & \Big\{\tfrac{1}{\varepsilon}\Big(\textstyle\int_{\Omega(\varepsilon)}\left(\chi_{E(\varepsilon)}W_1+(1-\chi_{E(\varepsilon)})W_2\right)(\nabla v)dx-\int_{\Omega(\varepsilon)}\bar{f}\cdot v\; dx\\ & +\alpha P(E(\varepsilon);\Omega(\varepsilon))\Big):\;\; v=0 \text{ on } \partial\omega\times(-\varepsilon,\varepsilon),\, \tfrac{1}{\mathcal{L}^3(\Omega(\varepsilon))}\textstyle\int_{\Omega(\varepsilon)}\chi_{E(\varepsilon)}\; dx=\lambda\Big\},\end{aligned} \tag{0.1}$$

where $\beta'(\Psi(|\xi|-1)\leq W_i(|\xi|)\leq\beta(1+\Psi(|\xi|))\;\;\forall\xi\in\mathbb{R}^{3\times3},\; i=1,2$, and some $\beta\geq\beta'>0$, while Ψ is an Orlicz convex function satisfying the ∇_2 and Δ_2 conditions. $E\,(\varepsilon)\subset\Omega\,(\varepsilon)$ is a measurable subset of $\Omega(\varepsilon)$ with finite perimeter. We assume that and the load $\hat{f}\in L^{\Psi^\star}(\Omega(\varepsilon);\mathbb{R}^3)$, where $\Psi^\star$ is the conjugate Orlicz function of Ψ.

This problem, commonly appearing in mechanical engineering, like the study of thin structures, can be investigated via functionals defined as follows. For every $\varepsilon>0$, let $J_\varepsilon: L^1(\Omega;\{0,1\})\times L^\Psi(\Omega;\mathbb{R}^3)\to[0,+\infty]$ we take

$$J_\varepsilon(\chi,u):=\begin{cases}\int_\Omega\left(\chi W_1\left(\nabla_{1,2}u\,\middle|\,\frac{1}{\varepsilon}\nabla_3 u\right)+(1-\chi)W_2\left(\nabla_{1,2}u\,\middle|\,\frac{1}{\varepsilon}\nabla_3 u\right)\right)dx & \\ -f\cdot udx+\alpha\left|\left(D_{1,2}\chi\,\middle|\,\frac{1}{\varepsilon}D_3\chi\right)\right|(\Omega) & \text{in } BV(\Omega;\{0,1\})\times W^{1,\Psi}(\Omega;\mathbb{R}^3),\\ +\infty & \text{otherwise,}\end{cases} \tag{0.2}$$

where $M|v$ is a matrix whose first two columns come from matrix M and the third one is vector v.

Our main result is calculating the Γ-limit of the family (0.2) with respect to the strong topology of $L^1(\Omega;\{0,1\})\times L^\Psi(\Omega,\mathbb{R}^3)$, when $\varepsilon\to0$.

The work is supported by Warsaw Center of Mathematics and Computer Science (KNOW) at University of Warsaw, http://wcmcs.edu.pl/.

Bogovski estimates and solenoidal difference quotients

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The Bogovski operator provides a solution to the divergence equation with a particular estimate of its gradient. In the talk, additional properties of the Bogovski solution will be presented, namely an estimate concerning difference quotients of the gradient. This information enables a construction of specific test functions with solenoidal (divergence-free) difference quotients. As an application, one gets a new way to prove interior regularity of the solution to the p-Stokes system. The used proof method relies particularly on the theory of singular integral operators and extrapolation.

Injective nonlinear elasticity via penalty terms: analysis and numerics

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I will present some new ideas for nonlinear elasticity with a global injectivity constraint preventing self-interpenetration of the elastic body. Our main focus are penalization terms replacing the injectivity constraint (the Ciarlet-Nečas condition). Among other things, the penalization can be chosen in such a way that self-interpenetration is prevented even at finite value of the penalization parameter, and not just in the limit. Our penalty method provides a working numerical scheme with provable convergence along a subsequence, for models of non-simple materials (including a term with higher order derivatives).

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Flow invariance of closed sets

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When studying the existence of solutions to systems of parabolic equations of the form $u_t + Lu = f(x, u)$, $u \in \mathbb{R}^M$, $x \in \Omega \subset \mathbb{R}^N$, subject to boundary conditions, where L is an elliptic (vector valued) differential operator and $f : \Omega \times \mathbb{R}^M \to \mathbb{R}^N$ is a continuous map, such that $u(x) \in C$ for a.a. $x \in \Omega$, where $C \subset \mathbb{R}^M$ is a given closed set of state constraints, an important hypotheses concern the so-called resolvent invariance of K, i.e. $(I + \lambda\mathbf{A})^{-\mathbf{1}}(\mathbf{K}) \subset \mathbf{K}$ for sufficiently small $\lambda > 0$, where (A) is the sectorial operator corresponding to L, $\mathbf{K} := \{u \in L^2(\Omega, \mathbb{R}^M) \,|\, u(x) \in C \text{ for a.a.} \, x \in \Omega\}$ (being equivalent to the invariance of $\mathbf{K}$ with respect to the semigroup generated by $\mathbf{A}$) and the so-called tangency of the nonlinear perturbation f. We will discuss the sufficient and necessary conditions for the invariance stated in terms of the coefficients of the operator L as well as in the language of Dirichlet bilinear form associated to L. This topic is strictly related to the study of the viability and invariance questions for partial differential equations.

This is a joint work with Jakub Siemianowski, Grzegorz Gabor and Aleksander Ćwiszewski.

Inverting of BV homeomorphisms

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This talk is devoted to the question of BV (or Sobolev) regularity of the inverse of a BV (or Sobolev) mapping. After a short review of the history we concentrate on most recent results. In [2], Stanislav Hencl, Rami Luisto and Aapo Kauranen have found sharp conditions for BV regularity of the inverse of a three-dimensional BV mapping. In the joint work [1] with Stanislav Hencl and Aapo Kauranen we establish the formula for the derivative of the inverse involving distributional adjugate of the gradient. The proofs are based on the degree formula for continuous planar BV mappings. This formula relates degree with the distributional Jacobian. The formula for the derivative of the inverse of a planar continuous BV mapping has been derived by Katarína Quittnerová in her unpublished master thesis (2007). In a joint work in progress with Luigi D'Onofrio, Carlo Sbordone and Roberta Schiattarella we prepare a stronger result on rectifiability of graph of such a mapping.

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Rayleigh–Bénard heat convection problem for the micropolar and Navier–Stokes fluid models

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We consider the Rayleigh–Bénard problem for the two and three-dimensional Boussinesq system for the micropolar fluid.

For the two-dimensional problem our main goal is to compare the value of the critical Rayleigh number, and estimates of the Nusselt number and the fractal dimension of the global attractor with those values for the same problem for the classical Navier–Stokes system. Our estimates reveal the stabilizing effects of micropolarity in comparison with the homogeneous Navier–Stokes fluid. In particular, the critical Rayleigh number for the micropolar model is larger then that for the Navier–Stokes one, and large micropolar viscosity may even prevent the averaged heat transport in the upward vertical direction. The estimate of the fractal dimension of the global attractor for the considered problem is better then that for the same problem for the Navier–Stokes system.

For the three–dimensional problem we introduce the notion of the multivalued eventual semiflow and prove the existence of the two-space global attractor $\mathcal{A}^K$ corresponding to weak solutions, for every micropolar parameter $K \geq 0$ denoting the deviation of the considered system from the classical Rayleigh–Bénard problem for the Newtonian fluid. We prove that for every K the attractor $\mathcal{A}^K$ is the smallest compact, attracting, and invariant set. Moreover, the semiflow restricted to this attractor is single-valued and governed by strong solutions. Further, we prove that the global attractors $\mathcal{A}^K$ converge to $\mathcal{A}^0$ upper semicontinuously in Kuratowski sense as $K \to 0$, and that the projection of $\mathcal{A}^0$ on the restricted phase space corresponding to the classical Rayleigh–Bénard problem is the global attractor for the latter problem, having the invariance property. These results are established under the assumption that the Prandtl number is relatively large with respect to the Rayleigh number.

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Atomic decompositions, two stars theorems, and distances for the Bourgain–Brezis–Mironescu space and other big spaces

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Given a Banach space E with a supremum-type norm induced by a collection of operators, we prove that E is a dual space and provide an atomic decomposition of its predual. We apply this result, and some results obtained previously by one of the authors, to the function space $\mathcal{B}$ introduced recently by Bourgain, Brezis, and Mironescu. This yields an atomic decomposition of the predual $\mathcal{B}_*$, the biduality result that $\mathcal{B}_0^* = \mathcal{B}_*$ and a formula for the distance from an element $f \in \mathcal{B}$ to $\mathcal{B}_0$. This is a joint project with L. Greco, K.M. Perfect, C. Sbordone ad R. Schiattarella.

Optimal local embeddings of Besov spaces involving only slowly varying smoothness

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My talk is based on a join work with Júlio Severino Neves (cf. [1]) and its aim is to mention optimal (local) embeddings of Besov spaces $B^{0;b}_{p;r}$ involving only a slowly varying smoothness b. In general, our target spaces are outside of the scale of Lorentz-Karamata spaces and are related to small Lebesgue spaces. In particular, we improve results from [2], where the targets are (local) Lorentz-Karamata spaces. To derive our results, we apply limiting real interpolation techniques and weighted Hardy-type inequalities.

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Multiplicity of positive solutions of nonlinear elliptic equations

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We deal with the existence of increasing sequences of positive solutions of the following class of nonlinear elliptic problems

$$\mathrm{div}(a(||x||)\nabla u(x)) + f(x, u(x)) + h(x, x \cdot \nabla u(x)) = 0,$$

where $x \in \mathbb{R}^n$ and $||x|| > R$, with the condition $\lim_{||x||\to\infty} u(x) = 0$. We apply an iteration scheme based on the subsolution and supersolution method. Our approach allows us to consider sublinear as well as superlinear problems without radial symmetry. Moreover we try to characterize more precisely the asymptotic behavior of solutions.

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Shrinking maps into Marcinkiewicz spaces are maximally noncompact

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We call a bounded linear map between two quasilinear spaces maximally noncompact if its measure of noncompactness coincides with its operator norm. We prove that a continuous but noncompact Sobolev embedding whose target is a Marcinkiewicz endpoint space is maximally noncompact even though such space is not disjointly superadditive. This is a joint work with Vít Musil of University of Florence, Italy, and Jan Lang of Ohio State University, Columbus, OH, USA.

A Neohookean model of plates

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We study hyperelastic deformations of neohookean materials in planar domains called plates. These problems are motivated by recent remarkable relations between Geometric Function Theory and the theory of Nonlinear Elasticity. Both theories are governed by variational principles. Here we confine ourselves to deformations of ℓ-connected bounded Lipschitz planar domains $\mathbb{X}$ and $\mathbb{Y}$. The general law of hyperelasticity tells us that there exists a stored energy function that characterizes the elastic and mechanical properties of the material. The subject of our investigation are Sobolev homeomorphisms $h \colon \mathbb{X} \twoheadrightarrow \mathbb{Y}$ with nonnegative Jacobian, $J_h := \det Dh \geqslant 0$; having smallest isotropic energy,

$$\mathcal{E}[h] = \int_{\mathbb{X}} E\left(|Dh|, \det Dh\right)\, dx, \quad \text{where} \quad E \colon \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}.$$

On the structure of smooth measures

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Consider the classical Dirichlet form on $L^2(D;dx)$, i.e. the form

$$\mathcal{E}(u,v)=\int_D \nabla u(x)\cdot\nabla v(x)\,dx,\quad u,v\in H_0^1(D).$$

In 1996 Boccardo, Gallouët and Orsina showed that a bounded (signed) Borel measure μ on D is absolutely continuous with respect to the capacity associated with $\mathcal{E}$ (i.e. the classical Newtonian capacity) if and only if μ is of the form

$$\mu=f+\nu$$

for some $f\in L^1(D;dx)$ and $\nu\in H^{-1}(D)$. In 2003 Droniou, Porretta and Prignet obtained a parabolic counterpart to this result. We will show that similar results hold for bounded Borel measures that are absolutely continuous with respect to the capacity associated with any (possibly nonlocal) regular Dirichlet form. Some applications of these results will be indicated.

My talk will be based on joint works [1]–[3] with Tomasz Klimsiak.

References

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Discontinuity of functions in Hajłasz space with critical exponent

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We consider a Hajłasz–Sobolev space $M^{1,s}$ on the metric space (X,d) equipped with a Borel measure μ. Recently, it has been showed by Zhou [4] that if $s\leq 1$, then functions in $M^{1,s}$ are uniformly continuous, provided that μ satisfies s-Ahlfors regularity condition. She has also conjectured that if $s>1$, then there is always in $M^{1,s}$ an example of discontinous function. I will present result from [1] in which we have given affirmative answer to Zhous hypothesis. Moreover, I will show an application of this result, comming from my master's thesis, which is generalization of the result from [2], i.e. we will discuss existence of closed, infinite dimensional subspace of $M^{1,s}$, which elements are nowhere bounded functions.

References

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Connecting orbits in Hilbert spaces and applications to PDE

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We prove a general theorem on the existence of heteroclinic orbits in Hilbert spaces, and present a method to reduce the solutions of some PDE. problems to such orbits. In our first application, we give a new proof in a slightly more general setting of the heteroclinic double layers (initially constructed by Schatzman [3]), since this result is particularly relevant for phase transition systems. In our second application, we obtain a solution of a fourth order PDE satisfying similar boundary conditions.

References

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Space optimality for kernel operators

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In this work, we study the behaviour of linear kernel operators on rearrangement-invariant (r.i.) spaces. In particular we focus on the boundedness of such operators between various function spaces. Given an operator and a domain r.i. space Y, our goal is to find an r.i. space Z such that the operator is bounded from Y into Z, and, whenever possible, to show that the target space is optimal (that is, the smallest such space). We concentrate on a particular class of kernel operators denoted by S_a, which have important applications and whose pivotal instance is the Laplace transform. It turns out that the problem of finding the optimal space for S_a can, to a certain degree, be translated into the problem of finding a "sufficiently small" space X such that a, the kernel of S_a, lies in X. If we do that we can construct inequalities, which allow us to essentially replace the integral containing function a with Calderón type operators.

Characterization of functions with zero traces from Sobolev spaces via the distance function from the boundary

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Consider a domain $\Omega \subset \mathbb{R}^N$ with Lipschitz boundary and let $d(x) = \operatorname{dist}(x, \partial\Omega)$. It is well known for $p \in (1, \infty)$ that $u \in W_0^{1,p}(\Omega)$ if and only if $u/d \in L^p(\Omega)$ and $\nabla u \in L^p(\Omega)$. Recently a new characterization appeared: it was proved that $u \in W_0^{1,p}(\Omega)$ if and only if $u/d \in L^1(\Omega)$ and $\nabla u \in L^p(\Omega)$. We found a more general condition on the function u/d, which together with $\nabla u \in L^p(\Omega)$ guarantees $u \in W_0^{1,p}(\Omega)$.

Reiterated homogenization in the Orlicz-Sobolev setting

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I will present some homogenization results for integral functionals obtained by means of two-scale convergence in the Orlicz–Sobolev setting. In particular some results in the higher order case and in the reiterated case will be shown.

References

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